Alternative Tile Assembly Models and Complexity Results

Tianqi Song
• All screenshot pictures are from original papers.
Outline

• Multiple Temperature model and complexity results.
• Staged assembly model and complexity results.
• Flexible glue model, time-dependent glue model and complexity results.
Multiple Temperature Model

- A tile system $S = \langle T, g, s, \{\tau_i\}_{i=1}^k \rangle$ under multiple temperature model [Aggarwal et al. SODA 2004].
- $\{\tau_i\}_{i=1}^k$ is temperature sequence.
- Temperature complexity: the number of items in the temperature sequence.
Assembly Process

• One pot reaction
• Multiple phases: each phase has its own temperature.
• Assemble and disassemble
Assembly process

Assemble and disassemble under temperature $\tau_1$ for long enough time.

Assemble and disassemble under temperature $\tau_k$ for long enough time.

The terminal product of phase $k$ is the terminal product of this k-temperature system.
Basic Tool

- Bit-Flipping [Kao et al. SODA 2006].

Figure 1: (a) Tiles that implement the bit flip gadget. The number of lines on the side of a tile represents the strength of the corresponding glue. The dark black line denotes a strength 5 glue. (b) At temperature 2, with tile $a$ as the seed tile, the bit flip gadget uniquely assembles a supertile with the 0 tile in the top right corner. (c) By raising the temperature to 5, the 0 tile breaks off and is replaced by the 1 tile.
Binary Strings with $O(1)$ Tiles

- Tile set [Kao et al. SODA 2006]:

![Diagram of Binary Strings with $O(1)$ Tiles]
Binary Strings with O(1) Tiles

• Example: 1010010
Binary Strings with $O(1)$ Tiles
Building $n \times n$ Square in $O(1)$ Tiles
Staged Assembly Model

• A tile system $S = <M_{r,b}, \{T_{i,j}\}, \{\tau_{i,j}\}>$ under staged assembly model[Demaine et al. Nat Comput 2008].

• Assumption: merge, split, extract

• $M_{r,b}$: r-stage b-bin mix graph $M$.

• $\{T_{i,j}\}$: $T_{i,j}$ is the tile set of bin$_{i,j}$.

• $\{\tau_{i,j}\}$: $\tau_{i,j}$ is the temperature of bin$_{i,j}$.

• Stage complexity, bin complexity
Example of Staged Assembly System

Figure 1: A sample staged assembly system that uniquely assembles a $1 \times 10$ line. The temperature is $\tau = 1$, and each glue $a, b, c$ has strength 1. The tile, stage, and bin complexities are 3, 3, and 2, respectively.
Assembly of $1\times n$ lines

• Special case: there is a planar temperature-1 staged assembly system that uniquely produce a (full connected) $1\times 2^k$ line using 3 glues, 6 tiles, 6 bins, and $O(k)$ stages [Demaine et al. Nat Comput 2008].

• Planar system:

Figure 2: All three assemblies are permitted under the basic model. However, only assembly (a) is permitted under the planarity constraint.
Assembly of $1 \times n$ lines

- Strategy: divide and conquer

Figure 3: Decomposition tree for $1 \times 16$ line.
Assembly of 1×n lines

• General case: standard trick, build 1×2\(^i\) line for the \(i\)th bit of binary encoding of \(n\) if the \(i\)th bit is 1.

• Theorem: there is a planar temperature-1 staged assembly system that uniquely produce a (full connected) 1×\(n\) line using 3 glues, 6 tiles, 7 bins, and \(O(\log n)\) stages [Demaine et al. Nat Comput 2008].
Assembly of $n \times n$ Squares

• Theorem: there is a planar temperature-1 staged assembly of a full connected $n \times n$ square using 9 glues, $O(1)$ tiles, $O(1)$ bins, and $O(\log n)$ stages[Demaine et al. Nat Comput 2008].
Figure 4: (a) The shifting problem encountered when combining rectangle supertiles. (b) The jigsaw solution: two supertiles that combine uniquely into a fully connected square supertile.
Decompose Algorithm

Algorithm DecomposeVertically (supertile $S$):

— Here $S$ is a supertile with $n$ rows and $m$ columns; $S$ is not necessarily a rectangle.

1. Stop vertical partitioning when width is small enough:
   If $m \leq 3$, DecomposeHorizontally($S$) and return.

2. Find the column along which the supertile is to be partitioned:
   Let $i := \lceil (m + 1)/2 \rceil$.
   Divide supertile $S$ along the $i$th column into a left supertile $S_1$ and right supertile $S_2$
   such that
   tiles at position $(1, i)$ and $(n, i)$ belong to $S_1$ and the rest of the $i$th column belongs
   to $S_2$.

3. Now decompose recursively:
   DecomposeVertically ($S_1$)
   DecomposeVertically ($S_2$)
Decompose Algorithm

Figure 5: Decomposition tree for $8 \times 8$ square in the jigsaw technique.
Figure 6: Assigning glues in the first two vertical decompositions of the jigsaw technique.
Flexible Glue Model

• Remove: \( g(x, y) = 0 \) for \( x \neq y \) [Aggarwal et al. SODA 2004].

• Assembly of \( N \times N \) square: the tile complexity of self-assembling \( N \times N \) squares is \( O(\sqrt{\log N}) \) under the flexible glue model.
Assembly of $k \times N$ Rectangles under Standard Model

- **Theorem:** the tile complexity of self-assembling a $k \times N$ rectangle is $O(N^{(1/k)}+k)$ for standard model [Aggarwal et al. SODA 2004].
- **Trick:** a $k$-digit base-$m$ counter, where $m=\text{ceil}(N^{(1/k)})$.
- **What if $N$ is not a power of $m$?** By initializing.
Assembly of $k \times N$ Rectangles under Standard Model

Figure 1: A tile set to assemble a $k \times m^k$ rectangle in $\Theta(k + m)$ tile complexity under the standard assembly model.
Assembly of $k \times N$ Rectangles under Standard Model

- C tiles: counting for the $0^{th}$ bit of the counter.
- H tiles: counting for the other bits of the counter.
- R,P tiles: transfer carry and flip bits back to 0.
Assembly of N×N Square under Flexible Glue Model

• Theorem: the tile complexity of self-assembling N×N squares is O(√logN) under the flexible glue model[Aggarwal et al. SODA 2004].

• Thought 1: the complexity of assembling N×N square is almost the complexity of assembling a (logN)-digit counter.

• Thought 2: how to assemble a (logN)-digit counter by O(√logN) tile types.
Assembly of $N \times N$ Square under Flexible Glue Model

- Thought 3: the complexity of assembling a counter is almost the complexity of assembling the first row of the counter.
- Thought 4: how to assemble the first row of a $(\log N)$-digit counter by $O(\sqrt[\log N]{\cdot})$ tile types.
- Target: Assemble the first row of a n-digit counter by $O(\sqrt[n]{\cdot})$ tile types.
Assembly of N×N Square under Flexible Glue Model

• Tile set:

Figure 4: This tile set creates a 2 × n block whose top row represents a given n-bit binary number b. Here $b_{ij}$ is the value of the bit in position $im + j$ in b. The glue function for glues $g_i^1$ and $g_j$ for $i$ from 0 to $m - 1$ and $j$ from 1 to $m - 2$ is $G(g_i^1, g_j) = b_{ij}$. All other pairs of non-equal glues have strength 0.
Assembly of $N \times N$ Square under Flexible Glue Model

• Example:

Figure 5: Assembling an arbitrary $n$-bit binary number in $O(\sqrt{n})$ tiles. Here we show the construction for $n = 36$ and binary number $b = \ldots 0110101110011010$. 
Time-dependent Glue Model

• The glue strength between glue x, y is a function of time t: $f_{x,y}(t)$[Sahu et al. DNA 2005].

Fig. 1 Figure illustrates the concept of time-dependent glue strength, minimum interaction time, and time for maximum strength.
Time-dependent Glue Model

- Minimum interaction time.
- Complexity result: the tile complexity of assembling a $k \times N$ rectangle is $O(\log N/\log \log N)$ under time-dependent glue model, where $k < \log N/ (\log \log N - \log \log \log N)$ [Sahu et al. DNA 2005].
- Trick: assemble a $j \times N$ rectangle where $j > k$. Disassemble the upper $(j-k)$ rows.