Test 2: CPS 100

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Declarations

For all problems on this test you can assume the following declarations have been made (sometimes a type other than int may be used for the `info` field).

```
struct Tree                    // "standard binary tree declaration"
{
    int info;
    Tree * left;
    Tree * right;
    Tree (int val, Tree * lchild = 0, Tree * rchild = 0)
    {
        info = val;
        left = lchild;
        right = rchild;
    }
};
```

PROBLEM 1 : (*Big-Oh Time:* 6 points)

Consider the following data structures:

- array in sorted order
- minheap (each node is smaller than its two children)
- binary search tree **(not necessarily balanced)**

and operations on the data structures:

- Find(x) returns true if x is present (in the structure), otherwise returns false
- FindMax returns the maximum element

Give the **worst case** running time (big-Oh) that best describes the running time of each operation.

	array - sorted order	minheap	binary search tree
Find(x)			
FindMax()			

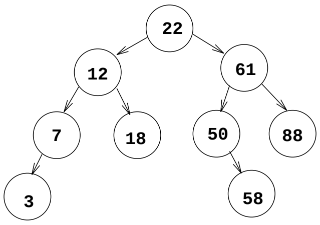
PROBLEM 2 : (*Lovelier than What?*)

Part A (*6 points*)

Draw the binary *search* tree that results from inserting the integers 23, 8, 40, 12, 32, 5, 15, 27 (in that order) into an initially empty tree.

Part B (*6 points*)

Give the preorder and postorder traversals of the tree shown below.



preorder:

postorder:

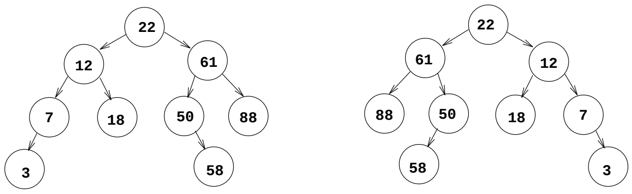
Part C (*2 points*)

The function below returns a copy of its tree parameter (See the Tree struct definition on page 2).

```
Tree * Copy(Tree * t)
// postcondition: returns copy of t
{
    Tree * temp = 0;
    if (t != 0)
    {
        temp = new Tree(t->info,
                        Copy(t->left),
                        Copy(t->right));
    }
    return temp;
}
```

What is the **average case** complexity of this function (using big-Oh notation)? Briefly justify your answer.

Part D (*4 points*) Indicate how to modify the function `Copy` to return a tree that is the mirror-image of the `Tree` parameter. The trees below are mirror images of each other (all left children in the mirror image are right children in the original tree and *vice-versa*).



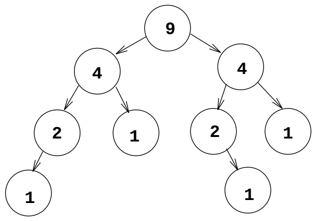
Part E (*4 points*)

Assume that a function *NumNodes* exists that returns the number of nodes in its `Tree` parameter.

```
int NumNodes(Tree * t)
// postcondition: returns number of nodes in t
```

The function `Copy` from Part C is modified so that the info field of each node in the copy returned is replaced by the count of the number of nodes in the tree rooted at `t` — the function is renamed `CopyCount`.

```
Tree * CopyCount(Tree * t)
// post: returns "counted" copy of t
{
    Tree * temp = 0;   // holds copy
    if (t != 0)
    {
        temp = new Tree(NumNodes(t),
                        CopyCount(t->left),
                        CopyCount(t->right));
    }
    return temp;
}
```



For example, if `t` is the tree in part B, the call `CopyCount(t)` returns the tree diagrammed above on the right.

The **average case** complexity of `CopyCount` is NOT $O(n)$; what is the complexity and why (briefly justify).

Part F (*10 points*)

Rewrite `CopyCount` so that it does have **average case** complexity $O(n)$. You may find it useful to use an auxiliary function called by `CopyCount`. The header for such an auxiliary function is shown below, you do NOT need to use this header but may. **You MUST indicate how your auxiliary function is called from** `CopyCount`.

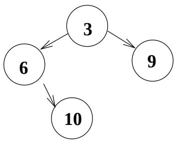
```
void CCAux(Tree * t, Tree * & copy, int & count)
```

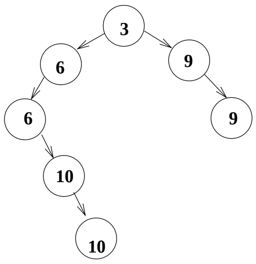
```
// postcondition: copy is a "copy count" copy of the tree t
//                e.g., all info fields replaced by node count
//                count is the number of nodes in tree t (rooted at t)
{




}
```

PROBLEM 3 : (*Double Vision:* 8 points)

Write the function *DoubleUp* whose header is given below. *DoubleUp* adds one new node to the tree `t` for every node except the root. For each non-root node, a new node is created with the same `info` field value and the same orientation (e.g., a left child or a right child). The newly created node will have only one child, that child will be the node that generates the newly created copy.

For example, if `t` is the tree on the left, `DoubleUp(t)` should modify `t` to look like the tree on the right.





Complete the function DoubleUp below. Use the Tree struct from page 2.

```
void DoubleUp(Tree * t)
// postcondition: modifies t so that it is "doubled up"
{
```

PROBLEM 4 : (*Recurring Recurrence:* 6 points)

Solve the following recurrence relation and give the **O()** that best describes the running time. Justify your answer.

$$T(1) = 1$$
$$T(n) = 2T(n/2) + n^2$$

PROBLEM 5 : (*ginorst stuff*)

Part A (*10 points*)

Suppose a struct `Student` for students in a class is (partially) declared as below. Each student has a name, a list of grades, the number of grades stored in the list, and a class identification number. In a class of N students, each student is given an ID number in the range $1 \dots N$, e.g., in a class of 20 students the numbers $1 \dots 20$ are assigned one per student.

```
struct Student
{
    int classID;                // ID number
    string name;                // name of student "Jane Doe"
    Vector<int> grades;         // list of grades
    int numGrades;              // # of items stored in grades
};
```

Write the function `SortStudents` whose header is given below. `SortStudents` sorts the information in `list` so that it is in order from smallest (1) to largest (N) student ID number.

Your solution MUST sort in O(N) time!!

(hint: in what slot does the student with ID number 16 belong?)

```
void SortStudents(Vector<Student> & list, int numElts)
// precondition: numElts = # of students in list
//               ID numbers are unique and in range 1...numElts
// postcondition: list is sorted into increasing order by
//                student ID number
// performance: O(n)
```

Part B (*10 points*)

You must solve the problem of determining if two numbers in an array of n numbers sum to a given number k. For example, if the array contains 8, 4, 1, 6, there are numbers that sum to 10 (4 and 6); to 9 (8 and 1); to 16 (use 8 twice); but NOT to 8 (exactly two numbers must be used).

Trying all pairs of numbers yields an $O(n^2)$ algorithm. Instead, you must describe how to:

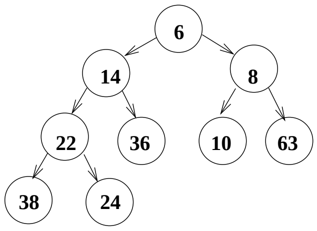
- implement an $O(n \log n)$ algorithm to solve this problem. You can use the array as storage, but only $O(1)$ other storage for this (e.g., int variables are fine, no vectors). Hint: given k (the number to sum to), if one of the two numbers is determined, the other one is known — you need to find if it's in the array by searching efficiently.
- implement an $O(n)$ algorithm (average case) by using auxiliary storage to facilitate an efficient search.

In both cases, describe how your algorithm will work (briefly) and justify the running time.

PROBLEM 6 : (*Heapless: EXTRA CREDIT* 6 points)

Complete the function *NumLessThan* to determine the number of elements in a minheap that are less than a *key*. A minheap is a heap such that the value in each node is less than the value in the node's subtrees. A minheap is implemented via a vector with the root at index 1.

In the example below, `NumLessThan(heap,9,13)` should return 3 (4, 10, and 6 are less than 13), and `NumLessThan(heap,9,8)` should return 1 (only 6 is less than 8).



The worst case running time of your function **should be O(k)** where k is the number of elements **less than** the key. (Note: If you want to use recursion, you might want to write an auxillary function.)

```
int NumLessThan(const Vector <int> & heap, int size, int key)
// precondition: size is the number of elements in the heap,
//              heap is a minheap
// postcondition: returns the number of elements in the heap
//                less than key
```