CPS 140 - Mathematical Foundations of CS
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Section: Parsing (handout)

Parsing

Parsing: Deciding if x$\in \Sigma^*$ is in L(G) for some CFG G.

Review

Consider the CFG G:

$$\begin{array}{l} S \rightarrow Aa \\ A \rightarrow AA \mid ABa \mid \epsilon \\ B \rightarrow BBa \mid b \mid \epsilon \end{array}$$

Is ba in L(G)? Running time?

Remove ϵ-rules, then unit productions, and then useless productions from the grammar G above. New grammar G' is:

$$\begin{array}{l} S \rightarrow Aa \mid a \\ A \rightarrow AA \mid ABa \mid Aa \mid Ba \mid a \\ B \rightarrow BBa \mid Ba \mid a \mid b \end{array}$$

Is ba in L(G)? Running time?

Top-down Parser:

- Start with S and try to derive the string.

$$S \rightarrow aS \mid b$$

- Examples: LL Parser, Recursive Descent

Bottom-up Parser:

- Start with string, work backwards, and try to derive S.

- Examples: Shift-reduce, Operator-Precedence, LR Parser

We will use the following functions FIRST and FOLLOW to aid in computing parse tables.

The function FIRST:

Some notation that we will use in defining FIRST and FOLLOW.

$$G=(V,T,R,S)$$
$$w,v \in (V \cup T)^*$$
$$a \in T$$
$$X,A,B \in V$$
$$X_I \in (V \cup T)^+$$

Definition: FIRST(w) = the set of terminals that begin strings derived from w.

If w $\stackrel{*}{\Rightarrow}$ av then
  a is in FIRST(w)
If w $\stackrel{*}{\Rightarrow}$ ϵ then
  ϵ is in FIRST(w)

To compute FIRST:

1. FIRST(a) = {a}
2. FIRST(X)
   (a) If X $\rightarrow$ aw then
   a is in FIRST(X)
   (b) IF X $\rightarrow$ ϵ then
   ϵ is in FIRST(X)
   (c) If X $\rightarrow$ Aw and $\epsilon \in$ FIRST(A) then
   Everything in FIRST(w) is in FIRST(X)
3. In general, FIRST($X_1X_2X_3..X_K$) =
   - FIRST(X_1)
   - $\cup$ FIRST(X_2) if ϵ is in FIRST(X_1)
   - $\cup$ FIRST(X_3) if ϵ is in FIRST(X_1)
     and ϵ is in FIRST(X_2)
     ...
   - $\cup$ FIRST(X_K) if ϵ is in FIRST(X_1)
     and ϵ is in FIRST(X_2)
     ... and ϵ is in FIRST(X_{K-1})
   - $-$ {ϵ} if $\epsilon \notin$ FIRST(X_J) for all J

Example: $L = \{a^n b^m c^n : n \geq 0, 0 \leq m \leq 1\}$

$$S \rightarrow aSc \mid B$$
$$B \rightarrow b \mid \epsilon$$

FIRST(B) =

FIRST(S) =

FIRST(Sc) =

Example

$$S \rightarrow BCD \mid aD$$
$$A \rightarrow CEB \mid aA$$
$$B \rightarrow b \mid \epsilon$$
$$C \rightarrow dB \mid \epsilon$$
$$D \rightarrow cA \mid \epsilon$$
$$E \rightarrow e \mid fE$$

FIRST(S) =

FIRST(A) =

FIRST(B) =

FIRST(C) =

FIRST(D) =

FIRST(E) =

Definition: FOLLOW(X) = set of terminals that can appear to the right of X in some derivation.

If S $\stackrel{*}{\Rightarrow}$ wAav then
a is in FOLLOW(A)

(where w and v are strings of terminals and variables, a is a terminal, and A is a variable)

To compute FOLLOW:

1. \$ is in FOLLOW(S)
2. If A $\rightarrow$ wBv and v $\neq \epsilon$ then
   FIRST(v) - $\{\epsilon\}$ is in FOLLOW(B)
3. IF A $\rightarrow$ wB OR
   A $\rightarrow$ wBv and ϵ is in FIRST(v) then
   FOLLOW(A) is in FOLLOW(B)
4. ϵ is never in FOLLOW

Example:

$$\begin{aligned}
&S \rightarrow aSc \mid B \\
&B \rightarrow b \mid \epsilon
\end{aligned}$$

FOLLOW(S) =

FOLLOW(B) =

Example:

$$\begin{aligned}
&S \rightarrow BCD \mid aD \\
&A \rightarrow CEB \mid aA \\
&B \rightarrow b \mid \epsilon \\
&C \rightarrow dB \mid \epsilon \\
&D \rightarrow cA \mid \epsilon \\
&E \rightarrow e \mid fE
\end{aligned}$$

FOLLOW(S) =

FOLLOW(A) =

FOLLOW(B) =

FOLLOW(C) =

FOLLOW(D) =

FOLLOW(E) =