CPS 140 - Mathematical Foundations of CS
Dr. S. Rodger
Section: Turing Machines (handout)

**Review**

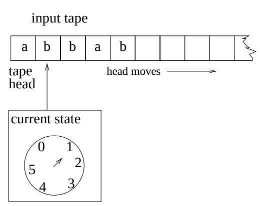
Regular Languages

- FA, RG, RE
- recognize

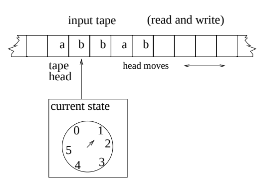
Context Free Languages

- PDA, CFG
- recognize

DFA:




Turing Machine:




**Turing Machine** (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

**Definition of TM**

- Storage
  - tape

- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    - start state
    - tape head on leftmost tape square
    - input string followed by blanks
  - processing computation
    - move tape head left or right
    - read from and write to tape
  - computation halts
    - final state

**Formal Definition of TM**

A TM M is defined by M=(K,$\Sigma$,, ,$\delta$,$q_0$,B,F) where

- K is finite set of states
- $\Sigma$ is input alphabet
- , is tape alphabet
- B$\in$ , is blank
- $q_0$ is start state
- F is set of final states
- $\delta$ is transition function
  $\delta$(q,a) = (p,b,R) means "if in state q with the tape head pointing to an 'a', then move into state p, write a 'b' on the tape and move to the right".

**TM as Language recognizer**

**Definition**: Configuration is denoted by $\vdash$.

if $\delta$(q,a) = (p,b,R) then a move is denoted

$$\text{abaqabba} \vdash \text{ababpbba}$$

**Definition:** Let M be a TM, M=(K,$\Sigma$,, ,$\delta$,$q_0$,B,F). L(M) = $\{w \in \Sigma^* | q_0 w \overset{*}{\vdash} x_1 q_f x_2$ for some $q_f \in$F, $x_1, x_2 \in$ , $^*\}$

**TM as language acceptor**

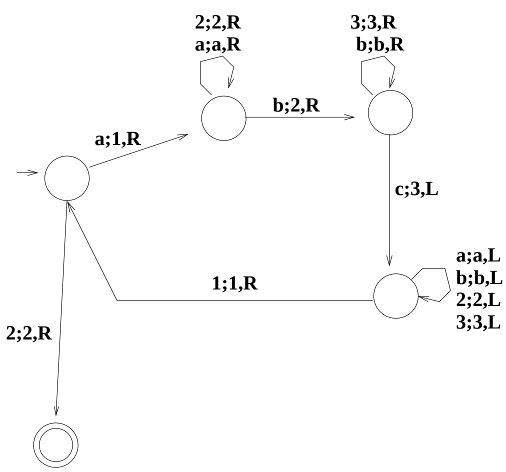
M is a TM, w is in $\Sigma^*$,

- if w$\in$ L(M) then M halts in final state
- if w$\notin$ L(M) then either
  - M halts in non-final state
  - M doesn't halt

**Example:**

L=$\{a^n b^n c^n | n \geq 1\}$

Is the following TM correct?




**TM as a transducer**

TM can implement a function: f(w)=w'

| | |
|---|---|
| start with: | w<br>$\uparrow$ |
| end with: | w'<br>$\uparrow$ |

**Definition:** A function with domain D is *Turing-computable* or *computable* if there exists TM M=(K,$\Sigma$, , ,$\delta$,$q_0$,B,F) such that

$$q_0 w \overset{*}{\vdash} q_f f(w)$$

$q_f$ $\in$F, for all $w$ $\in$D.

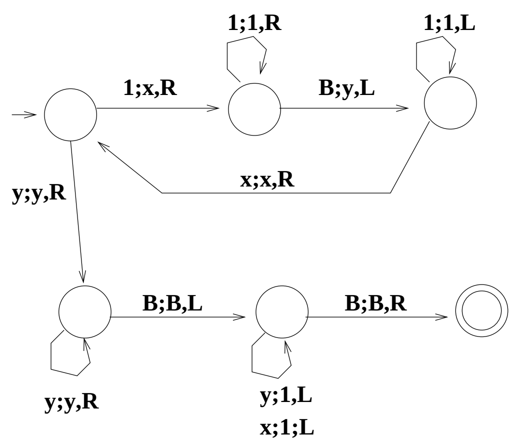
**Example:**

f(x) = 2x

x is a unary number

| | |
|---|---|
| start with: | 111<br>↑ |
| end with: | 111111<br>↑ |

Is the following TM correct?




**Example:**

L=$\{ww \mid w \in \Sigma^+\}$, $\Sigma=\{a, b\}$