Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:

Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
• actions
  - write symbol
  - read symbol
  - move left (L) or right (R)

• computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM M is defined by M=(K,Σ, δ,q₀,B,F) where

• K is finite set of states
• Σ is input alphabet
• is tape alphabet
• B∈ is blank
• q₀ is start state
• F is set of final states
• δ is transition function

δ(q,a) = (p,b,R) means “if in state q with the tape head pointing to an ‘a’, then move into state p, write a ‘b’ on the tape and move to the right”.

TM as Language recognizer

Definition: Configuration is denoted by ⊩.

if δ(q,a) = (p,b,R) then a move is denoted

abaqabba ⊩ ababpbba

Definition: Let M be a TM, M=(K,Σ, δ,q₀,B,F). L(M) = \{ w ∈ Σ* | q₀w ⊩ x₁qⱼx₂ for some qⱼ ∈ F, x₁,x₂ ∈ Σ* \}
TM as language acceptor
M is a TM, w is in \( \Sigma^* \),

- if \( w \in L(M) \) then M halts in final state
- if \( w \notin L(M) \) then either
  - M halts in non-final state
  - M doesn’t halt

Example:
\( L=\{a^n b^n c^n | n \geq 1 \} \)

Is the following TM correct?

TM as a transducer
TM can implement a function: \( f(w) = w' \)

Definition: A function with domain D is Turing-computable or computable if there exists TM M=(K, \( \Sigma \), \( \delta \), \( q_0 \), B, F) such that

\[
q_0 w \xrightarrow{\ast} q_f f(w)
\]

\( q_f \in F \), for all \( w \in D \).
Example:

$f(x) = 2x$

$x$ is a unary number

<table>
<thead>
<tr>
<th>start with:</th>
<th>end with:</th>
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<tbody>
<tr>
<td>111</td>
<td>111111</td>
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Is the following TM correct?

Example:

$L=\{ww \mid w \in \Sigma^+\}, \Sigma=\{a, b\}$