1. (10 pts) Complete the following.
   (a) \(\{3, 2\} \times \{a, b\} = \)
   (b) \(\{a^n b^m \mid n > 0\} \cup \{a^n b^m \mid n \geq 0, m \geq 0\} = \)
   (c) \(\Sigma = \{a, b\}, \{w \in \Sigma^* \mid w \text{ has an even number of } a's \} \cap b^* a^* = \)
   (d) \(2^{[a]} = \)
   (e) \(L_1 = \{0, 3, 6, 9, 12, 15, 18, 21, \ldots\}, L_2 = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, \ldots\}, \overline{L_1 \cup L_2} = \)

2. (12 pts) Answer TRUE or FALSE to each of the statements below.
   (a) \(\emptyset \in \emptyset\)
      (TRUE or FALSE?)
   (b) \(L = \{a^n b^m c^p \mid 0 < n < 100, 0 < m < 200, p = n + m\}. \) \(L\) is regular.
      (TRUE or FALSE?)
   (c) \(L = \{b^n (aa)^m \mid m > n > 0\}. \) \(L\) is regular.
      (TRUE or FALSE?)
   (d) \(\Sigma = \{a, b\}, L = \{w \in \Sigma^* \mid n_a(w) \text{ mod } 5 = 1 \text{ and } n_b(w) < n_a(w)\}. \) \(L\) is regular.
      (TRUE or FALSE?)
   (e) \(G\) is a CFG such that \(\epsilon \in L(G). \) There exists a regular grammar \(G'\) such that \(L(G) = L(G'). \)
      (TRUE or FALSE?)
   (f) \(G\) is a CFG with 2 rules. Then there exists a DFA \(M\) such that \(L(G) = L(M). \)
      (TRUE or FALSE?)

3. (3 pts) Give an example of \(L_1\) and \(L_2\) such that \(L_1\) is regular, \(L_2\) is NOT regular and \(L_1 \cap L_2\) is regular.

4. (8 pts) The reverse of a string is defined as follows where \(a \in \Sigma, \) and \(w \in \Sigma^*.\)
   \(a^R = a\)
   \((wa)^R = aw^R\)
   Prove by induction \((uv)^R = v^R u^R\) for all \(u, v \in \Sigma^*.\) (Show all steps: the basis, induction hypothesis (I.H.) and induction step (I.S.).)

5. (12 pts) Draw DFA’s (not NFA’s!) for the following languages. Do not show trap states. (For each transition diagram, indicate the start state by a short arrow, and final states by double circles.)
(a) \( L = \{w \in \Sigma^* \mid w \text{ has both } ab \text{ and } ba \text{ as a substring}\}, \Sigma = \{a, b\}. \\
(b) \ L = \{w \in \Sigma^* \mid n_a(w) \mod 3 = 1 \text{ and there is exactly } 1 \ b\}, \Sigma = \{a, b\}. \\

6. (8 pts) Convert the following NFA to a DFA using the algorithm discussed in class.

[Diagram of NFA]

7. (8 pts) Convert the following DFA to a minimum state DFA. Show the tree distinguishing the states and explain at each level the reason for distinguishing the states. **Show the resulting minimal DFA** with states labeled with names from their original states (State AB would be combined states A and B).

[Diagram of DFA]

8. (9 pts) Consider the following DFA M.

[Diagram of DFA]

(a) The formal definition of this DFA is \( M = (K, \Sigma, \delta, q_0, F) \), where
i. \( K = \{ \} \)

ii. \( \Sigma = \{ \} \)

iii. \( F = \{ \} \)

(b) Give a **regular expression** \( R \) such that \( L(R) = L(M) \).

9. (6 points) Write a **regular grammar** for \( L = \{ w \in \Sigma^* \mid n_b(w) \text{ mod } 3 = 2 \} \), \( \Sigma = \{ a, b \} \)

10. (8 points) Consider the following language. \( L = \{ a^n b^m c^p \mid m > n + p, n > 0, p > 0 \} \)

   (a) Write a **context-free grammar** for \( L \).

   (b) Show the parse tree for \( aabbbbc \).

**Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with

\[
\begin{align*}
|xy| & \leq m \\
|y| & \geq 1 \\
x y^i z & \in L \quad \text{for all } i \geq 0
\end{align*}
\]

11. (8 pts) Use the Pumping Lemma to prove

   \( \Sigma = \{ a, b, c \} \), \( L = \{ w \in \Sigma^* \mid n_a(w) * n_b(w) > n_c(w) \} \) is not regular.

   **Proof:** (SHOW ALL STEPS! Some have been started for you.)

   Assume

   Choose \( w = \)

12. (8 pts) Consider the following property, SwapEnd.

   \( \Sigma = \{ a, b, c, d \} \) SwapEnd\((L) = \{ wba | waba \in L, w \in \Sigma^* \} \).

   The property SwapEnd applied to a language \( L \) accepts only those strings from \( L \) that end in \( aab \), but first replaces the \( aab \) at the end of the string by \( ba \). For example, if the string \( babaab \) is in \( L \), then \( babba \) is in SwapEnd\((L) \). If \( aabaab \in L \), then \( aabba \in \) SwapEnd\((L) \).

   **Prove** that the regular languages are closed under the SwapEnd property. (Show all steps!)