TERRASTREAM: From Elevation Data to Watershed Hierarchies*

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ABSTRACT

We consider the problem of extracting a river network and a watershed hierarchy from a terrain given as a set of irregularly spaced points. We describe TERRASTREAM, a "pipelined" solution that consists of four main stages: construction of a digital elevation model (DEM), hydrological conditioning, extraction of river networks, and construction of a watershed hierarchy. Our approach has several advantages over existing methods. First, we design and implement the pipeline so that each stage is scalable to massive data sets; a single non-scalable stage would create a bottleneck and limit overall scalability. Second, we develop the algorithms in a general framework so that they work for both TIN and grid DEMs. Furthermore, TERRASTREAM is flexible and allows users to choose from various models and parameters, yet our pipeline is designed to reduce (or eliminate) the need for manual intervention between stages.

We have implemented TERRASTREAM and we present experimental results on real elevation point sets, which show that our approach handles massive multi-gigabyte terrain data sets. For example, we can process a data set containing over 300 million points—over 20GB of raw data—in under 26 hours, where most of the time (76%) is spent in the initial CPU-intensive DEM construction stage.

Categories and Subject Descriptors: F.2 [Theory of Computation]: Analysis of Algorithms and Problem Complexity

General Terms: ALGORITHMS, DESIGN, EXPERIMENTATION

Keywords: Terrain modeling

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1 Introduction

Recent revolutionary improvements in mapping technologies are rapidly expanding the impact of Geographic Information Systems (GIS). In particular, laser altimetry (lidar) gathers georeferenced *elevation* data as a set of points in $\mathbb{R}^3$ at unprecedented resolutions and rates. While lidar enables critically important applications, such as environmental and disaster management, many technical challenges make the development of these applications difficult. One of the main challenges is to develop robust and efficient algorithms for terrain modeling and analysis that can handle massive data sets.

In this paper we consider the problem of extracting a river network and a watershed hierarchy from a set of points in $\mathbb{R}^3$ sampled from a terrain. Intuitively, a river network is a collection of paths that indicate where water flow accumulates and creates well defined channels. A watershed hierarchy is a hierarchical partition of the terrain into connected regions, or *watersheds*, where all water within a region flows toward a single common outlet. We describe TERRASTREAM, a scalable solution that consists of a sequence of algorithms that form a pipeline. Each algorithm in the pipeline scales to massive data sets. Our pipeline is flexible and allows users to choose from various models and parameters, with no or minimal manual intervention between stages. We also present experimental results on real lidar data using TERRASTREAM that demonstrate its scalability.

1.1 Background and previous results

Terrain modeling and analysis. Typically, a terrain in a GIS is not stored as a set of points, but rather as a digital elevation model (DEM), either in the form of a *Triangulated Irregular Network* (TIN) or a *grid*. In a TIN DEM, the terrain is represented by a planar triangulation, where each vertex has an associated elevation; in a grid DEM, it is represented as a two dimensional array of points, where each grid point represents an elevation. Both DEM formats are common in many GIS applications.

Terrain modeling and analysis has been studied extensively in many different communities, and algorithms have been developed

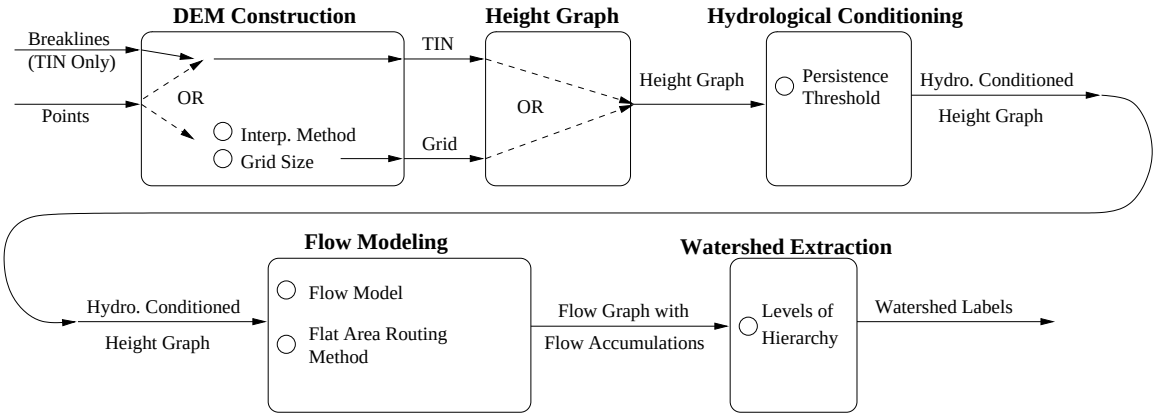


Figure 1. Overview of pipeline stages showing inputs, outputs, and optional modeling parameters for each stage.

for many fundamental problems. Refer to [32] and the references therein for a survey. Many GIS applications use a *pipeline* (or *work-flow*) approach for combining many smaller, simpler algorithms into a larger, more complex application. Often, the individual stages in a pipeline are developed independently and require manual intervention to pre-process or post-process the data between stages. Furthermore, while a typical GIS can manage gigabytes of data consisting of hundreds or thousands of smaller individual data sets, most systems are not designed to handle single multi-gigabyte data sets. Moreover, previous GIS algorithms designed to handle massive data sets have focused on individual stages of the pipeline, and have been designed for only either grid or TIN DEMs.

I/O-efficient algorithms. The massive terrain data sets we consider can be much larger than the main memory of typical machines and must reside on large but slow disks. In such cases the transfer of data between disk and main memory, not CPU computation time, often becomes the primary bottleneck. Therefore we are interested in designing efficient algorithms in the I/O-model [4]. In this model, the machine consists of a main memory of size M and an infinite-size disk. A block of B consecutive elements can be transferred between main memory and disk in one *I/O operation* (or simply *I/O*). Computation only occurs on elements in main memory, and the complexity of an algorithm is measured in terms of the number of I/Os it performs. Many fundamental problems have been solved in the I/O model. For example, sorting N elements requires $\text{SORT}(N) = \Theta(\frac{N}{B} \log_{M/B} \frac{N}{B})$ I/Os. Refer to surveys by Vitter [31] and Arge [5] for other results.

Only recently have terrain problems been considered in the I/O-model. I/O-efficient algorithms have been developed for construction of either TIN or grid DEMs from a set of input points [1, 2, 18, 19], as well as for certain water flow problems, including river network extraction, on grid DEMs [8]. The I/O-efficient flow algorithms for grid DEMs have been distributed in the TERRAFLOW software package [8]. Very recently, an I/O-efficient algorithm for extracting watershed hierarchies from a grid DEM river network has also been developed and implemented [9]. These algorithms typically use $O(\text{SORT}(N))$ I/Os.

1.2 Our results

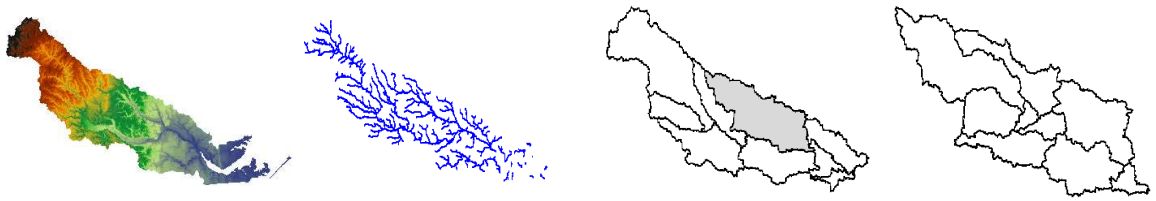
In contrast to earlier approaches, TERRASTREAM is a pipeline of I/O-efficient algorithms and their implementation that automatically computes a watershed hierarchy from a point set $\mathcal{S}$ in $\mathbb{R}^3$. Our pipeline is highly efficient, scalable, modular, and flexible. It scales to single multi-gigabyte sized data sets, works for both grid and TIN DEMs, and is faster than other scalable algorithms currently implemented, e.g., TERRAFLOW [8] in GRASS [24]. The highly modular and configurable pipeline is designed to reduce manual intervention, and to allow for easy addition of new modeling features. Our approach provides several parameters to control the behavior of each pipeline stage, and users can choose between several popular models in each stage.

TERRASTREAM consists of four main stages: DEM construction, hydrological conditioning (sink removal), flow modeling including extraction of river networks, and extraction of watershed hierarchies. Figures 1 and 2 illustrate the overall structure of the pipeline and the outputs of its stages. TERRASTREAM builds upon and extends a number of previously developed I/O-efficient terrain algorithms, and includes several new algorithms designed to complete a whole pipeline. A considerable amount of engineering effort has been devoted to making TERRASTREAM efficient and practical. Our main technical contributions in this paper include the following:

- We take a unified approach for handling both TIN and grid DEMs. We represent TIN and grid DEMs as a graph, which we refer to as a *height graph*. We then design algorithms in the subsequent pipeline stages to use height graphs. Our methods therefore work on both grid and TIN DEMs. Such a unified approach makes it easier to maintain software and to add new features. Moreover, the unified approach does not come at a cost of decreased performance. Our pipeline works on a given DEM type as efficiently as if the code were written solely for that particular type.

- We implement an $O(\text{SORT}(N) \log(N/M))$-I/O algorithm for assigning a numerical score or *significance* to each sink, or local minimum, in a height graph. We then use a sink's significance for *hydrologically conditioning*—we remove insignificant sinks from a terrain while preserving significant sinks such as large closed depressions with no outlet. This step of removing unimportant sinks is crucial to standard flow models.

- In addition to extending earlier grid based flow modeling algorithms [8] to height graphs, we develop a simple and practical algorithm for detecting flat areas in a terrain. We also implement an improved algorithm for flow routing on flat areas. Flat areas commonly cause problems in flow modeling algorithms. Flat areas may exist in either the original data



(a) (b) (c) (d)

Figure 2. (a) DEM of Neuse river basin derived from lidar points (b) Rivers with drainage greater than 5000 acres (2023 hectares) extracted from DEM. (c) First level of Pfafstetter watershed labels for largest basin in Neuse. (d) Recursive decomposition of the shaded basin.

due to insufficient vertical resolution or be introduced into the terrain as a side-effect of hydrological conditioning.

The rest of the paper is organized as follows. We briefly describe the main stages of our pipeline in Sections 2–5. In Section 6 we present a number of experimental results on real lidar data that demonstrate the power of our pipeline. For example, we can process a data set containing over 300 million points—over 20GB of raw data—in under 26 hours; most of the time (76%) is spent in the initial CPU-intensive DEM construction stage and can be reduced using simpler interpolation schemes [18, 19]. We also show that the relevant portions of TERRASTREAM are significantly faster than the corresponding TERRAFLOW [8] algorithms.

2 DEM Construction

The first stage of our pipeline constructs a grid or TIN DEM from a set $\mathcal{S}$ of N input points in $\mathbb{R}^3$. Below we briefly review the algorithms we utilize; the reader is referred to [1, 2] for a complete overview of, and comparison with, previous work. We also introduce the the notion of a height graph, which we use in later stages.

Grid DEM construction. The common approach for constructing a grid DEM given a user-specified grid resolution from $\mathcal{S}$ is to use one of many interpolation or approximation methods to compute a height value for each grid point (refer to e.g., [22] and the references therein). For inputs with more than a few thousand points, applying an interpolation method directly on all points is infeasible because of the computational complexity of solving large systems of linear equations. Instead we chose for TERRASTREAM a recently developed I/O-efficient algorithm [1] that uses a quadtree segmentation in combination with a regularized spline with tension interpolation method [23]. It constructs a grid DEM in $O(\frac{N}{B}\frac{h}{\log\frac{M}{B}} + \text{SORT}(T))$ I/Os, where h is the height of a quad tree on $\mathcal{S}$ and T is the number of points in the desired grid DEM. Note that the algorithm uses $O(\text{SORT}(N) + \text{SORT}(T))$ I/Os if $h = O(\log N)$, that is, if the points in $\mathcal{S}$ are distributed such that the quad tree is roughly balanced. Our implementation is modular and allows users to implement a variety of interpolation methods (instead of the regularized spline method). The spline method we use allows smooth approximation of data, and can therefore accurately compute properties such as slope, profile curvature, and tangential curvature (which are important for landform analysis and landscape process modeling). We store the output grid in a simple row-major format to allow efficient grid row access in later stages.

TIN DEM construction. The most popular method for constructing TINs from elevation points is to project the points onto the xy-plane, compute their Delaunay triangulation, and then lift the triangulation back to 3D. In many GIS terrain processing applications, however, elevation data sets are often supplemented with line segments or *breaklines* that provide additional elevation information along linear features such as roads or rivers. Breaklines constrain the edges of the TIN to match breakline segments and preserve important topological features. In TERRASTREAM, we use a randomized I/O-efficient algorithm [2] for constructing a *constrained Delaunay triangulation* [11] of a set $\mathcal{S}$ of N points and a set $\mathcal{L}$ of K line segments, where all K line segments appear as edges of the final triangulation. The algorithm uses $\text{SORT}(N)$ expected I/Os if the number of constraining segments K is smaller than then memory size M. In most applications, K is considerably smaller than both N and M. We store the output TIN in an "indexed triangle" format, which is a common, simple, and compact representation of TINs. In this format, the coordinates of the TIN vertices are stored consecutively on disk along with a unique vertex ID, followed by a list of triangles each identified by three vertex IDs in clockwise order.

Height graph. To avoid designing separate grid and TIN algorithms for each of our successive pipeline stages, we define a graph, which is typically referred to as a *height graph*, that unifies both DEM formats. A height graph $G = (V, E)$ is an undirected graph derived from a DEM, with a *height* $h(v)$ and an *id* $id(v)$ associated with each $v \in V$. The id's are assumed to be unique. For any pair of vertices u and v, we say that u is *higher* than v if $h(u) > h(v)$, or if $h(u) = h(v)$ and $id(u) > id(v)$. The concept of *lower* than is defined similarly. The vertices and edges of a TIN DEM naturally form a height graph. To construct a height graph from a grid DEM, we include all the grid points as the vertices of the graph. For each grid point u, we add edges from u to some of its eight immediate neighbors.

In both the TIN and the grid case, we add an additional "outside" vertex ξ with $h(\xi) = -\infty$, which is connected to all the vertices on the boundary of the DEM. A height graph can be constructed from a grid or TIN DEM of size N in $O(\text{SORT}(N))$ I/Os.

3 Hydrological Conditioning

Most flow modeling algorithms assume water will flow downhill until it reaches a local minimum or *sink*. In practice, local minima in DEMs fall into two primary categories; *significant* and *insignificant* (or spurious) sinks. Significant sinks correspond to large real geographic features such as quarries, sinkholes or large natural *closed* depressions with no drainage outlet. The insignificant sinks correspond to noise in the input data or to small natural features that flood easily. When modeling water flow, these insignificant sinks impede flow and result in artificially disconnected hydrological networks. The second stage of our pipeline "hydrologically conditions" a DEM for the flow modeling stage by removing insignificant sinks, while preserving significant sinks.

A widely used hydrological conditioning algorithm removes *all* sinks using a so-called *flooding* approach [20], which simulates

(a) (b) (c)

Figure 3. (a) Original terrain. (b) Terrain flooded with $\tau = \infty$. (c) Terrain partially flooded with persistence threshold $\tau = 30$.

uniformly pouring water on the terrain until a steady-state is reached. A weakness of this approach is that it removes even significant sinks. See Figures 3(a) and (b). Furthermore, the previous I/O-efficient algorithm [8] for hydrological conditioning works only for grids and assumes that all sinks fit in memory. This assumption does not hold for large high-resolution terrains. We instead use a *partial flooding* algorithm, based on *topological persistence* [16, 15], that detects and removes only insignificant sinks, as indicated in Figure 3(c). We briefly describe topological persistence and then present our algorithm.

Topological persistence. In the context of a terrain T represented by a planar height graph, topological persistence [16, 15], matches each local minimum (sink) vertex v of T to a higher "saddle" vertex w (see [14] for the precise definition of a saddle) and assigns a *persistence* value, denoted by $\pi(v)$, to v. In [15], $\pi(v)$ is defined to be the difference in the heights of v and w, i.e., $\pi(v) = h(w) - h(v)$. The persistence $\pi(v)$ denotes the *significance* of the sink v. Intuitively, the saddle w is a vertex at which two distinct connected components of the portion of T lying strictly below w merge. Suppose each connected component is represented by the lowest vertex in the component, and that v is the higher representative of two connected components merged by w; let u denote the representative of the other component. Then topological persistence induces a *merge tree* on the sinks of T, in which u is the parent of v. The merge tree has the property that the heights of vertices on any root-to-leaf path increase, while the persistence values decrease along such a path.

Agarwal et al. [3] developed an $O(\text{SORT}(N))$-I/O algorithm for computing the persistence of all sinks in a triangular planar height graph, as well as the merge tree. They also developed and implemented a simpler and practical $O(\text{SORT}(N)\log(N/M))$-I/O algorithm. We extend the latter to form our partial flooding algorithm given below.

Partial flooding. We use topological persistence as a measure of the significance of a sink. Given a user-specified threshold τ, we declare all sinks with persistence less than τ to be the insignificant sinks and remove all such sinks using a partial flooding method described below. The user can change the threshold to control the smallest feature size to be preserved.

We define partial flooding of a height graph by generalizing the flooding definition for grid DEMs [20, 8]. Let G be a height graph with significant sinks $\zeta_1, \ldots, \zeta_k$. Let the *height of a path* in G be the height of the highest vertex on the path, and let the *raise elevation* of a vertex v of G be the minimum height of all paths from v to ζ_i for any $1 \leq i \leq k$. In *partial flooding*, we change the height of each vertex in G to its raise elevation. Partial flooding produces a modified height graph containing only significant sinks whose persistence value is greater than τ. Note that if $\tau = \infty$, our definition of partial flooding is the same as the original definition of flooding. Thus, partial flooding is a tunable way to condition the terrain for the purpose of flow modeling.

To efficiently condition a terrain using partial flooding, we utilize the following property of the merge tree whose proof can be found in [13]: Let u be a vertex in the merge tree that does not correspond to a significant sink, but whose parent does. Let v be any vertex in the sub-tree rooted at u. Then the raise elevation $r(v)$ of v is $r(v) = r(u) = h(u) + \pi(u)$. This allows us to compute the raise elevations for each sink in the merge tree (or more precisely, the sinks of G corresponding to vertices in the merge tree) in a simple way: For each insignificant sink u in the merge tree whose parent corresponds to a significant sink, we propagate $r(u)$ to all vertices below u. To do so efficiently we simply direct tree edges from a vertex to its children and traverse the vertices in height order while forwarding the relevant raise elevation along outgoing edges. This traversal can be performed in $O(\text{SORT}(N))$ I/Os using standard techniques [12, 6].

What remains is to compute the raise elevations for all non-sink vertices in the height graph G. To do so we assign a sink label to each vertex in G. A vertex u is assigned sink label v if there is a path of monotonically decreasing height from u to a sink v; if several such paths exists, we choose the one to the lowest sink v. The raise elevation of u is then simply $r(u) = \max\{h(u), r(v)\}$ [13]. To assign the labels efficiently to all vertices, we construct a DAG by directing edges in G from lower height vertices to higher height vertices. The vertices in this DAG are naturally sorted in topological order by increasing height. We traverse the DAG in topological order and forward sink labels along outgoing edges; the sink label for a vertex u is simply the label corresponding to the lowest sink among the labels received from the preceding vertices. This traversal is similar to the merge tree-traversal and can be performed in $O(\text{SORT}(N))$ I/Os [12, 6].

In summary, for a given threshold τ, we can partially flood the terrain represented as a height graph in $O(\text{SORT}(N))$ I/Os.

4 Flow Modeling

4.1 Flow routing and accumulation

The third stage of our pipeline models the flow of water on a hydrologically conditioned DEM, represented as a height graph. It consists of two phases. In the first *flow-routing* phase, we compute a *flow direction* for each vertex v in the height graph that intuitively indicates the direction water will flow from v. In the second *flow-*

accumulation phase, we intuitively compute the area of the terrain represented by vertices upstream of each vertex v.

Flow routing. Given a height graph $G = (V, E)$, the flow-routing phase computes a directed subgraph $\mathcal{F}(G) = (V, E_r)$ of G called the *flow graph*. An edge (v, u) in $\mathcal{F}(G)$ indicates that water can flow from v to u. We say that u is *downstream neighbor* of v and v is *upstream neighbor* of u. In general, we say v is *upstream* of a vertex w if there is a path from v to w in $\mathcal{F}(G)$. E_r is constructed from G by looking at each vertex v and its neighbors and applying a *flow-direction* model. We implemented two popular flow-direction models:

- *Single-flow-direction* (SFD) model: for each vertex v, the edge from v to the neighbor with lowest height lower than the height of v is selected.
- *Multi-flow-directions* (MFD) model: for each vertex v, all edges from v to neighbors of lower height are selected.

Several other flow-direction models have also been proposed (e.g., [28, 21]), and most of them can be incorporated in our pipeline. We refer the reader to [8] for more information on SFD and MFD routing. If the height of every vertex in G is distinct, we can easily construct $\mathcal{F}(G)$ in $O(\text{SORT}(N))$ I/Os using standard techniques, by simply examining the neighbors of every vertex in the height graph and assign a flow directions to all but the sinks. In the SFD and MFD models, the resulting flow graph is a forest or a DAG, respectively. Terrain models, however, can have large *flat areas* of vertices with no neighbors of lower height. Flat areas can be natural plateaus in the terrain model, or they can appear as by-products of the flooding. Detecting these flat areas and routing flow through them in a realistic way is challenging, and we discuss these steps further in Section 4.2. We have implemented extensions of SFD and MFD models that incorporate routing on flat areas.

Flow accumulation. Given a flow graph $\mathcal{F}(G)$ with flow directions, the flow accumulation [26] phase intuitively computes the area of the terrain represented by vertices upstream of each vertex v. More precisely, each vertex v in the flow graph $\mathcal{F}(G)$ is assigned some initial flow. Each vertex then receives incoming flow from upstream neighbors and distributes all incoming and initial flow to one or more downstream neighbors. The flow accumulation of a vertex v is the sum of its initial flow and incoming flow from upstream neighbors.

Following the above definition, our flow accumulation algorithm simply visits the vertices of $\mathcal{F}(G)$ in topological order and for each vertex v, computes the total incoming flow and distributes flow to each downstream neighbor u with an edge (v, u) in $\mathcal{F}(G)$ using a given function. In our implementation we distribute flow in proportion to the height difference between v and u, but our pipeline allows other distribution functions. Our algorithm is a slight generalization of a $O(\text{SORT}(N))$ I/O algorithm by [10] developed for grid DEMs. In terms of initial flow, one typically assigns a "unit" of initial flow to each vertex if G represents a grid DEM, since each grid vertex represents an area of the same size. If G represents a TIN DEM, one typically distributes the xy-projection of the area of each triangle in G equally among its three vertices. We have implemented these choices, but TERRASTREAM also allows for the user to specify an initial flow for each vertex.

Given the flow accumulations for all vertices, we can extract *river networks* [26] in $O(\text{SORT}(N))$ I/Os, simply by extracting edges incident to vertices whose flow accumulation exceeds a given threshold.

4.2 Handling flat areas

A robust flow model must handle extended flat areas in a terrain . A vertex v in a height graph G is *flat* if $h(v) \leq h(u)$ for all neighbors u of v in G, or if v has a neighbor of the same height that has no lower neighbors. A *flat area* is a maximal connected component of flat vertices of the same height. A *spill point* of a flat area is a flat vertex with a downstream neighbor. Routing flow on flat areas is composed of two steps; detecting all flat areas and routing flow across each individual flat area.

Detecting flat areas. Detecting flat areas is equivalent to finding connected components of same-height vertices in G. Although a previous theoretical $O(N/B)$-I/O algorithm for computing connected components on grid DEMs [10] exists, it is too complex to be of practical interest. Furthermore, it can not be extended to work on height graphs. We developed and implemented a simpler algorithm for height graphs that scans the vertices and their neighbors and uses a batched union-find structure to merge vertices in the same flat area into a single connected component. We used a simple and practical union-find implementation [3] such that the algorithm uses $O(\text{SORT}(N) \log(N/M))$ I/Os.

Detecting flat areas on a grid DEM. Since TERRASTREAM is modular and allows us to plug in customized modules easily, we have also implemented a simplified algorithm for detecting flat areas on grid DEMs that uses $O(N/B)$ I/Os assuming that a constant number of rows (or columns) of the grid fit in memory. In this case, we can, in practice, handle grid DEMs containing several terabytes of space using only 256 MB of main memory.

Intuitively, our algorithm performs two row-by-row sweeps of the grid DEM, while only keeping two grid rows and a small union-find structure in main memory, and assigns every vertex in the same connected flat area the same unique *connected component label*. The union-find structure maintains connected component labels for the two grid rows currently in memory. The first sweep is a *down-sweep* from the topmost to bottommost row in the grid that assigns provisional connected component labels to each flat vertex. After the down-sweep all flat vertices with the same label are in the same connected component. However, a single flat area may have multiple labels. We therefore perform a second *up-sweep* from the bottommost to topmost row in the grid that assigns a single unique connected component label to all vertices in the same flat area. The sweeps are described in detail below.

In the down-sweep, we keep the current row and the row immediately above it in memory. In the top row, each flat vertex has already been assigned a connected component label. To process the current row we first visit the vertices in the row from left to right and assign a new unique label $l(u)$ to each flat vertex u. Then we visit each vertex u in the current row again and perform a UNION on $l(u)$ and $l(v)$ for any pair of neighboring flat vertices (u, v) currently in memory. We implement the union-find structure such that the unique representative for a set of labels is the label that was assigned earliest. Finally, we update the label of each flat vertex u in the current row to be the label FIND(u). We can prove that after processing the current row, two vertices in the same flat area in the current row have the same label if and only if they are connected by a path completely contained in the current row and the rows above it [13]. We can also show that all vertices on the bottom-most row of a flat area have the same label as the first label assigned in the highest row [13]. At the end of the up-sweep this will be the unique label assigned to the flat area.

In the up-sweep, we also keep two rows in memory; the current

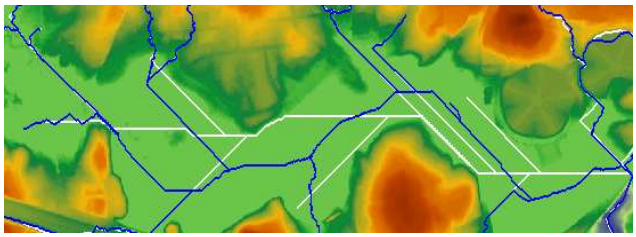


Figure 4. Comparison of routing methods on a flat area with a single spill point on the right. Rivers indicated in white were extracted by using the smallest Euclidean distance. Black river lines were computed using the Soille et al. approach.

row and the row immediately below it. To process the current row we first visit the vertices in the row from left to right and determine for each flat vertex u if it has a flat neighbor v in the row below the current row; if so we perform a UNION on $l(u)$ and $l(v)$. As in the down-sweep, we then update the label of each flat vertex u in the current row to be the label FIND(u). We can prove that after the up-sweep, vertices in the same connected flat area have the same connected component label [13].

Since the union-find structure used during the two sweeps never contains more labels than there are vertices in two rows, we can implement it such that it uses space proportional to the space occupied by a row. Thus it fits in main memory at all times and does not require any I/Os. Therefore our algorithm uses $O(N/B)$ I/Os, because it only scans the grid DEM twice.

Improved routing on flat areas. When routing flow on flat areas we distinguish between flat areas that have at least one spill point and those with no spill point. In the first case water should be able to flow out of the flat area through the spill points, while in the second case water is simply absorbed into the extended sink.

Many earlier approaches to flat area routing (see e.g. [8] and references therein) assign flow in a simple way such that each vertex v are assigned a flow direction to the neighbor on the shortest (Euclidean) path edge from v to the closest spill point. However, these approaches are not hydrologically realistic and tend to create many parallel flow lines [29]. Recently, a new more realistic flat area routing approach was proposed by Soille et al. [27]. Their approach, based on geodesic time and distance, improves an earlier, popular approach by Garbrecht and Martz [17]. Given a flat area, define H to be the set of flat vertices having an upstream neighbor. The algorithm of Soille et al. [27] computes the minimum distance d_v from each of the other flat vertices v to a vertex in H. Let $d_{\max}$ be the maximum distance d_v computed in the flat area. Each vertex v is assigned a flow direction to the first vertex on the minimum-cost path from v to a spill vertex, where the cost of a path is defined as the sum of $d_{\max} - d_u$ for all vertices u along the path. If no spill vertex exists, the minimum cost paths from a vertex with distance $d_{\max}$ is used. Since $d_{\max} - d_u$ is large near the upstream boundaries, the shortest paths will converge toward the low cost vertices away from the boundaries. This substantially increases the convergence of the flow routing paths.

We implemented both the Soille et al. [27] approach, and a simple shortest path approach. Figure 4 compares the two. Both approaches are implemented under the assumption that each flat area fits in main memory; our experience with high resolution elevation data indicates this is a reasonable assumption.

5 Watershed Hierarchy Extraction

Given a flow graph $\mathcal{F}(G)$ in which each vertex in $\mathcal{F}(G)$ is augmented with its flow accumulation, the final stage of our pipeline computes a watershed hierarchy for G. As mentioned earlier, a watershed hierarchy is a hierarchical decomposition of a terrain into a set of disjoint regions, or watersheds, where all water flows towards a single outlet. Such a decomposition is the basis of several GIS algorithms for hydrological and pollutant transport modeling.

Verdin and Verdin [30] described a method that divides a terrain into nine disjoint watersheds and thereafter recursively subdivide each of these watersheds. The terrain is thus hierachally divided into arbitrarily small regions. In the process each vertex is assigned a unique *Pfafstetter label* that encodes the watershed it belongs to on each level of the hierarchy, as well as topological properties such as upstream and downstream ordering.

Arge et al. [9] developed an algorithm using $O(\text{SORT}(N) + T/B)$ I/Os for computing the Pfafstetter labels of a grid DEM; here T is the total size of the labels. The algorithm uses a data structure equivalent to a flow graph $\mathcal{F}(G)$ computed using a single flow direction model and augmented with flow accumulations for each vertex. We modified the algorithm to use the flow graph $\mathcal{F}(G)$, that is, to work for flow graphs derived from a height graph.

6 Experiments

We have implemented TERRASTREAM in C++ using TPIE [7], a library that provides support for implementing I/O-efficient algorithms and data structures. Figure 1 gives an overview of the pipeline inputs, options, and outputs; note that each stage in the pipeline can also be used independently on a grid or TIN DEM. As mentioned in the Introduction, TERRASTREAM is highly modular and designed to reduce manual intervention while providing several parameters to control the behavior of each pipeline stage and allowing new models and features to be added with minimal effort. We highlight only a few key features in this extended abstract; additional details can be found at `http://terrain.cs.duke.edu` or in [13]. We have experimented extensively with TERRASTREAM on multiple data sets but, for lack of space, present only a limited set of experimental results that demonstrate the practicality and scalability of the pipeline. Again we refer the reader to [13] for more extensive experiments.

Experimental setup. We performed experiments on a Dell Precision Server 370 (3.40 GHz Pentium 4 processor) running Linux 2.6.11. The machine had 1 GB of physical memory, though our experiments never required more than 640 MB. All test data was stored on a single 400 GB SATA disk drive.

To demonstrate the scalability on a real watershed, we used a collection of 477 million bare Earth lidar points (over 20GB of raw data) from the Neuse river basin in North Carolina as our main test data. This data is publicly available for download from the North Carolina flood mapping project [25] and covers an area of roughly 6,200 square miles (16,000 km^2). The average point spacing in the set is approximately 20 feet (6m).

Pipeline scalability. We present three experiments on the Neuse river basin data set to illustrate the pipeline scalability. In the first experiment we construct a TIN DEM in the first stage of the pipeline, while we construct 10ft and 20ft grid DEMs in the last two experiments. Running times for each of the pipeline stages in the three experiments are shown in Table 1. A visual overview of the output of the different stages for the 20ft grid case is shown in Figure 2.

Format	20 ft grid	10 ft grid	TIN
# of height graph vertices (millions)	397	1590	469
Pipeline stage			
DEM Construction	19h 56m	27h 12m	4h 20m
Building height graph	0h 07m	0h 30m	11h 42m
Hydrological conditioning	1h 17m	7h 25m	10h 03m
Flow Modeling			
Flow Routing	1h 26m	6h 34m	15h 08m
Flow Accumulation	1h 40m	7h 35m	2h 05m
Watershed extraction	2h 28m	14h 39m	6h 26m
Total	25h 54m	63h 34m	49h 44m

Table 1. Running times for various pipeline stages (and sub-stages) on the Neuse river basin data set.

As seen in Table 1, the TIN DEM construction stage is much faster than the grid DEM construction stage since the former does not involve a sophisticated interpolation routines (which solve large systems of linear equations). In fact, the DEM construction stage is by far the most time consuming stage in the grid DEM experiments; as noted in [1], more than half of the total grid construction stage running time is spent on performing CPU-intensive interpolation. Note that the construction time for the 10ft grid DEM is not significantly longer than that of the 20ft grid, despite the fact that the latter has four times as many vertices. The reason is that the running time of our grid construction algorithm is more heavily influenced by the number of input points used in the interpolation than by the number of grid vertices (output points). Furthermore, the number of input points to the 20ft grid interpolation algorithm is actually smaller (but not 4 times smaller) than the number of input point to the 10ft grid interpolation algorithm (339 and 415 million, respectively) because the construction algorithm discards points that are close to each other relative to the grid cell size.

After constructing a DEM from the input points, our pipeline constructs a height graph. As seen from Table 1, this step is much more costly in the TIN case than in the grid case. This is because the output of the TIN construction algorithm is a set of triangles without any connectivity information, while the grid DEM construction algorithm returns a two-dimensional array (with implicit connectivity information).

All the remaining stages of our pipeline work on a height graph and therefore their running time should theoretically only depend on the number of input vertices (and edges). In the grid case we observe that each of the stages for the 10ft grid takes roughly four times as long time as for (the four times smaller) 20ft grid. This is to be expected since $\text{SORT}(N)$ does not grow much faster than linearly for similarly sized inputs. However, we also observe that the hydrological conditioning and flow routing steps take much more time for the TIN DEM than for the 20ft grid DEM of comparable size. The reason is that, as in the construction stage, we can take advantage of the implicit connectivity information in the case of a grid DEM. Finally, the flow routing stage on the TIN DEM is also much slower than for the 20ft grid DEM. The reason is obviously that we use the simple flat area detecting algorithm described in Section 4 in the grid case, while we must use a more complicated I/O-efficient connected component algorithm [3] in the TIN case.

Comparison with TERRAFLOW. For grid DEMs, TERRAFLOW [8] provides the same functionality as the portion of our pipeline from building the height graph through computing flow accumulation, provided that we configure our hydrological conditioning stage to remove *all* sinks. We therefore also compared the running time of TERRASTREAM to the running time of TERRAFLOW on the 20ft grid. TERRASTREAM finished in 4.5 hours, while TERRAFLOW finished after 12.2 hours. The hydrological conditioning (flooding) stage of TERRAFLOW was particular slow at 6 hours, while TERRASTREAM needed only 1.28 hours. There are two primary reasons for our speedup over TERRAFLOW. First, TERRAFLOW uses a different algorithm that also has a $O(\text{SORT}(N))$ I/O bound, but performs more scanning and sorting steps to compute the raise elevations. Second, by using edges and vertices of $\mathcal{F}(G)$ directly during flow accumulation, we have a compact representation for vertex connectivity. In contrast, each vertex in TERRAFLOW keeps a copy of all eight neighbors regardless of height when computing the accumulation, effectively multiplying the original input size by nine.

Hydrological conditioning persistence values. As mentioned, our hydrological conditioning stage allows us to tune a persistence threshold in order to remove insignificant sinks. As one final illustration of the features and properties of our software pipeline, we consider the distribution of sink persistence values in the Neuse river basin dataset.

There were 12.5 million sinks in the 20ft grid DEM, roughly 3% of the height graph vertices. However, over 94% of these sinks had a persistence value of less than 1ft (30cm), and 99.9% of all sinks had a persistence value of less than 6ft. There were only 15 sinks with a persistence greater than 50ft (15m) and all but one of these corresponded to quarries; the last (with a persistence value of approximately 50ft) was due to a bridge crossing a steep river valley. The sinks with the 100 highest persistence values had persistence value greater than 29.7ft. By visual inspection, we found that most of these sinks were due to bridges crossing waterways. We also found that the top 100 sinks for the TIN and 10ft grid had similar, but not identical, persistence values as compared to the 20ft grid. Typically the differences were less than 1ft. The 10ft grid had 27.3 million (about 1.7% of all height-graph vertices) sinks while the TIN had the most sinks at 32.8 million or 6.8% of all vertices. The number of sinks is higher in the TIN DEM because unlike the grid DEM, no smoothing via approximation or interpolation was performed. This illustrates one advantage of the expensive interpolation/approximation step performed in the construction of the grid DEM.

Overall, we found that a persistence of 50ft resulted in a well connected hydrological network while preserving most significant sinks.

7 Future Work

In this paper we described TERRASTREAM, a pipeline of scalable algorithms and their implementations that extract river networks

and a watershed hierarchy from a set of elevation data points.

We are currently extending TERRASTREAM in many ways: other interpolation schemes for grid DEMs, which are not as computationally intensive as the one currently used; more sophisticated methods for removing insignificant sinks in the hydrological conditioning stage; and building a hierarchical representation of a DEM that preserves river networks.

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